EE 435

Lecture 3
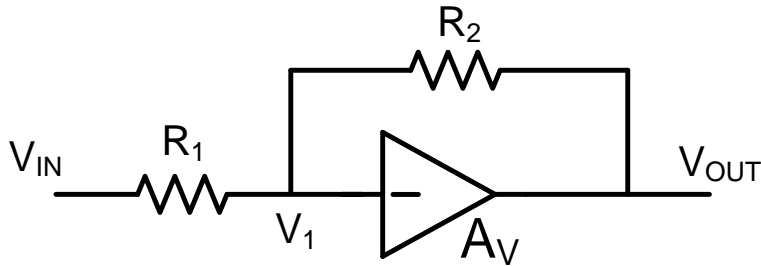
Design Space Exploration

with applications to single-stage amplifier design

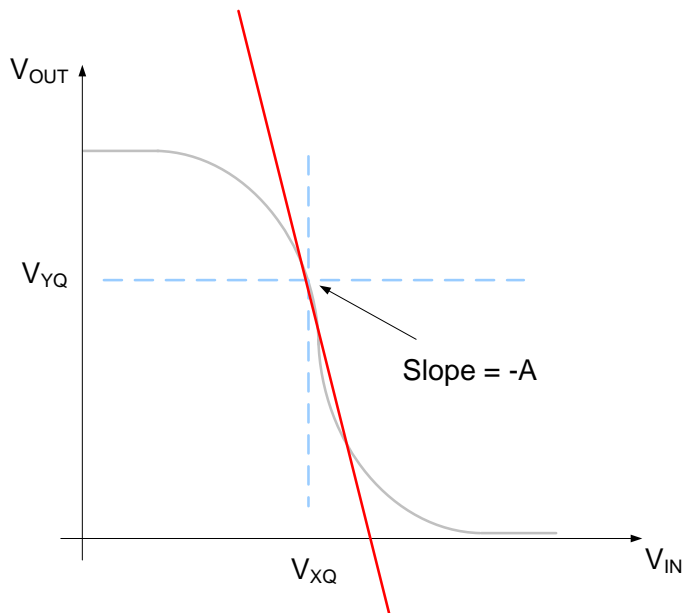Systematic Strategies for Designing and Analyzing Op Amps

Single-ended Op Amp Inverting Amplifier

$$V_O = (-A)(V_1 - V_{XQ}) + V_{YQ}$$

$$V_1 = \frac{R_1}{R_1 + R_2} V_O + \frac{R_2}{R_1 + R_2} V_{IN}$$

Summary:

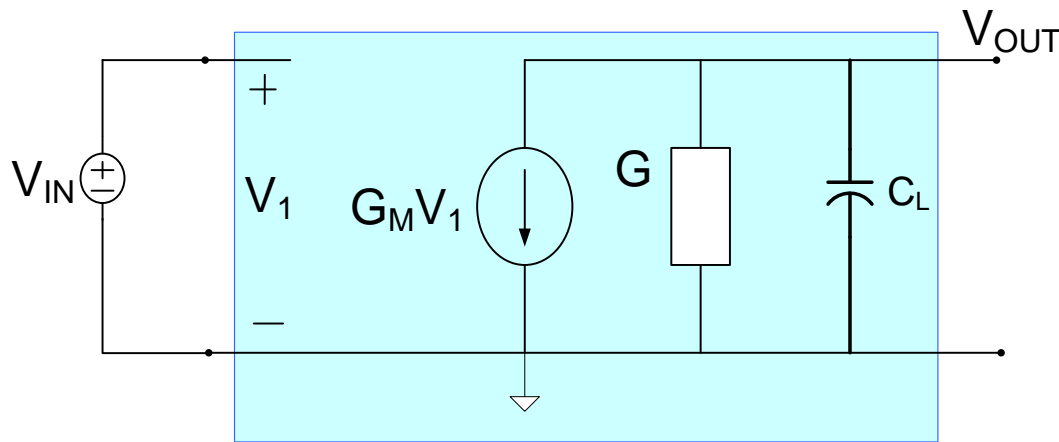$$V_O = -\frac{R_2}{R_1} V_{iss} + V_{XQ} + \frac{R_2}{R_1}(V_{XQ} - V_{inQ})$$

What type of circuits have the transfer characteristic shown?

4

Single-stage single-input low-gain op amp
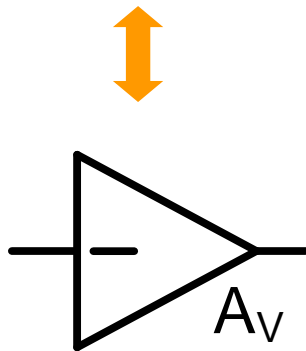(unilateral with infinite input impedance and one capacitor)

$$A_V = \frac{-G_M}{sC_L + G}$$

$$A_{v0} = \frac{-G_M}{G}$$

$$BW = \frac{G}{C_L}$$

$$GB = \left(\frac{G_M}{G}\right)\left(\frac{G}{C_L}\right) = \frac{G_M}{C_L}$$

GB and A_{VO} are two of the most important parameters in an op amp

How do we design an amplifier with a given architecture in general or <u>this architecture in particular</u>?

What is the design space?

Generally V_{SS}, V_{DD}, C_L (and possibly V_{OUTQ}) will be fixed

Must determine { W_1, L_1, I_{DQ} and V_{INQ} }

Thus there are 4 design variables

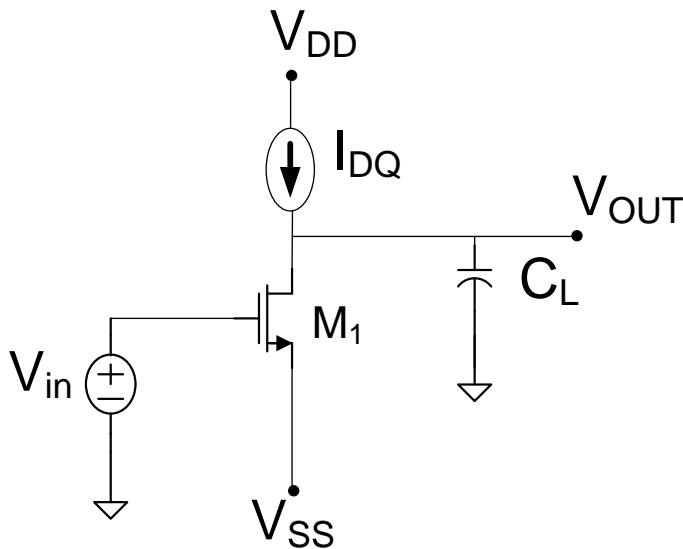But W_1 and L_1 appear as a ratio in almost all performance characteristics of interest

and I_{DQ} is related to V_{INQ}, W_1 and L_1
(this is a constraint)

Thus the design space generally has only two independent variables or two degrees of freedom $\left\{\dfrac{W_1}{L_1}, I_{DQ}\right\}$

Thus design or "synthesis" with this architecture involves exploring the two-dimensional design space $\left\{\dfrac{W_1}{L_1}, I_{DQ}\right\}$

V_{DD}

I_{DQ}

V_{OUT}

C_L

M_1

V_{in}

V_{SS}

6

How do we design an amplifier with a given architecture ?

1. Determine the design space

2. Identify the constraints

3. Determine the entire set of unknown variables and the Degrees of Freedom

4. Determine an appropriate parameter domain

5. Explore the resultant design space with the identified number of Degrees of Freedom

Parameter Domains for Characterizing Amplifier Performance

Degrees of Freedom: 2

Small signal parameter domain : $\{g_m, g_0\}$

$$A_{V0} = \frac{-g_m}{g_0} \qquad GB = \frac{g_m}{C_L}$$

Natural design parameter domain: $\left\{\dfrac{W}{L}, I_{DQ}\right\}$

$$A_{V0} = \left[\frac{\sqrt{2\mu C_{OX}}}{\lambda}\right]\left[\frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}}\right] \qquad GB = \left[\frac{\sqrt{2\mu C_{OX}}}{C_L}\right]\left[\sqrt{\frac{W}{L}}\sqrt{I_{DQ}}\right]$$

Alternate parameter domain: $\{P, V_{EB}\}$

$$A_{V0} = \left[\frac{2}{\lambda}\right]\left[\frac{1}{V_{EB}}\right] \qquad GB = \left[\frac{2}{V_{DD}C_L}\right]\left[\frac{P}{V_{EB}}\right]$$

Process Dependent

8

Parameter Domains for Characterizing Amplifier Performance

Degrees of Freedom: 2

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$$A_{V0} = \frac{-g_m}{g_0} \qquad GB = \frac{g_m}{C_L}$$

Natural design parameter domain: $\quad \left\{\dfrac{W}{L}, I_{DQ}\right\}$

$$A_{VO} = \left[\frac{\sqrt{2\mu C_{OX}}}{\lambda}\right]\frac{\sqrt{\frac{W}{L}}}{\sqrt{I_{DQ}}} \qquad GB = \left[\frac{\sqrt{2\mu C_{OX}}}{C_L}\right]\sqrt{\frac{W}{L}}\sqrt{I_{DQ}}$$

Alternate parameter domain: $\qquad\qquad \left\{P, V_{EB}\right\}$

$$A_{V0} = \left[\frac{2}{\lambda}\right]\left[\frac{1}{V_{EB}}\right] \qquad GB = \left[\frac{2}{V_{DD}C_L}\right]\left[\frac{P}{V_{EB}}\right]$$

Architecture Dependent

9

Parameter Domains for Characterizing Amplifier Performance

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Alternate parameter domain: $\left\{P, V_{EB}\right\}$

$$A_{V0} = \left[\frac{2}{\lambda}\right]\boxed{\left[\frac{1}{V_{EB}}\right]} \qquad GB = \left[\frac{2}{V_{DD}C_L}\right]\boxed{\left[\frac{P}{V_{EB}}\right]}$$

- **Alternate parameter domain gives considerable insight into design**
- **Easy to map from alternate parameter domain to natural parameter domain**
- **Alternate parameter domain provides modest parameter decoupling**
- $A_{V0}\left[\dfrac{\lambda}{2}\right]$ **and** $GB\left[\dfrac{V_{DD}C_L}{2}\right]$ **figures of merit for comparing architectures**
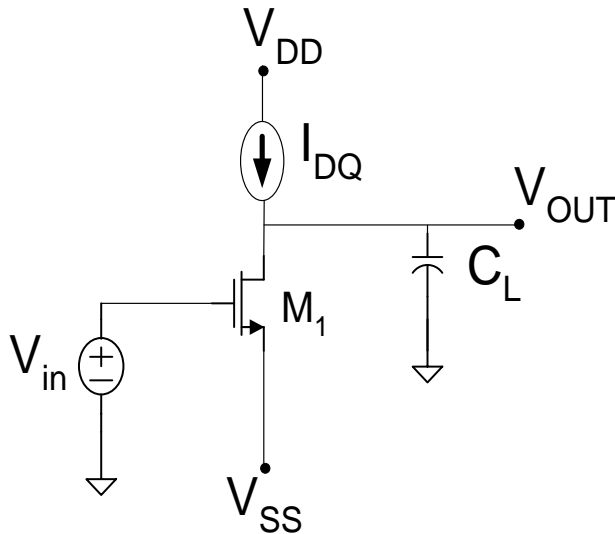
Design With the Basic Amplifier Structure

Consider basic op amp structure

Alternate parameter domain: $\{P, V_{EB}\}$

Degrees of Freedom: 2

$$A_{V0} = \left[\frac{2}{\lambda}\right]\left[\frac{1}{V_{EB}}\right]$$

$$GB = \left[\frac{2}{V_{DD}C_L}\right]\left[\frac{P}{V_{EB}}\right]$$

$$I_{DQ} = \frac{P}{V_{DD}} \qquad \frac{W}{L} = \frac{P}{V_{DD}\mu C_{OX} V_{EB}^2} \qquad V_{INQ} = V_{SS} + V_T + \sqrt{I_{DQ}\frac{2}{\mu C_{OX}}\frac{L}{W}}$$

But what if the design requirement dictates that $V_{INQ} = 0$?

Question: How can one meet two or more performance requirements with one design degree of freedom with this circuit?

Degrees of Freedom: 1

Luck or Can't

How do we design an amplifier with a given architecture ?

1. Determine the design space

2. Identify the constraints

3. Determine the entire set of unknown variables and the Degrees of Freedom

4. Determine an appropriate parameter domain
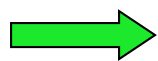
5. Explore the resultant design space with the identified number of Degrees of Freedom

Design Space Exploration

Question: How does the GB of the single-stage amplifier common-source amplifier change with bias current?

$$GB = \frac{g_m}{C_L} \qquad \longrightarrow \qquad GB = \left[\frac{2}{C_L}\right]\left[\frac{I_{DQ}}{V_{EB}}\right]$$
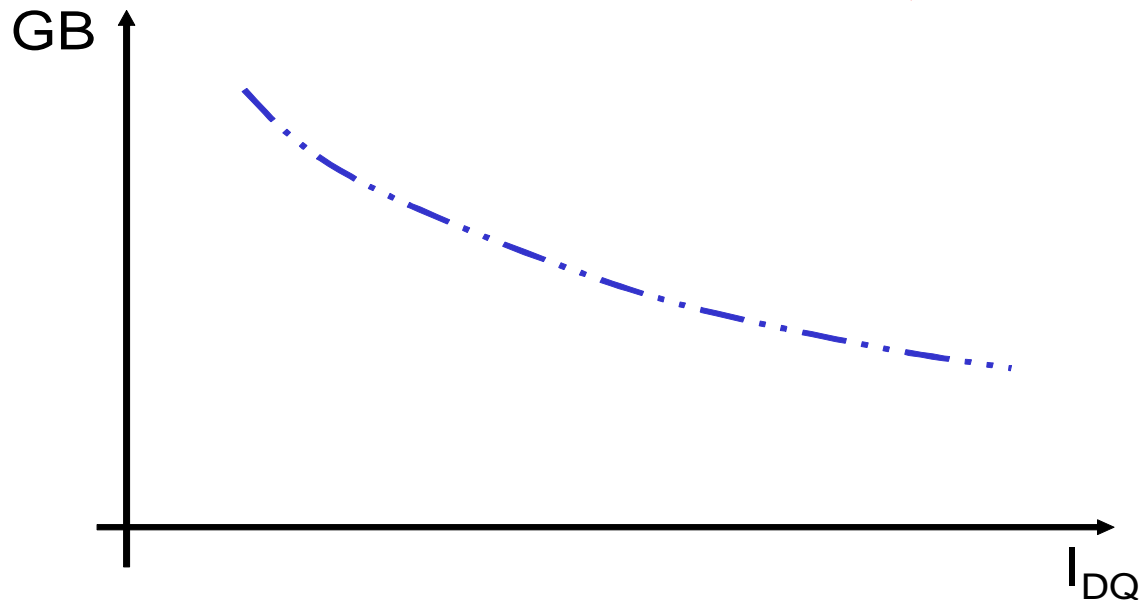
GB increases linearly with I_{DQ}
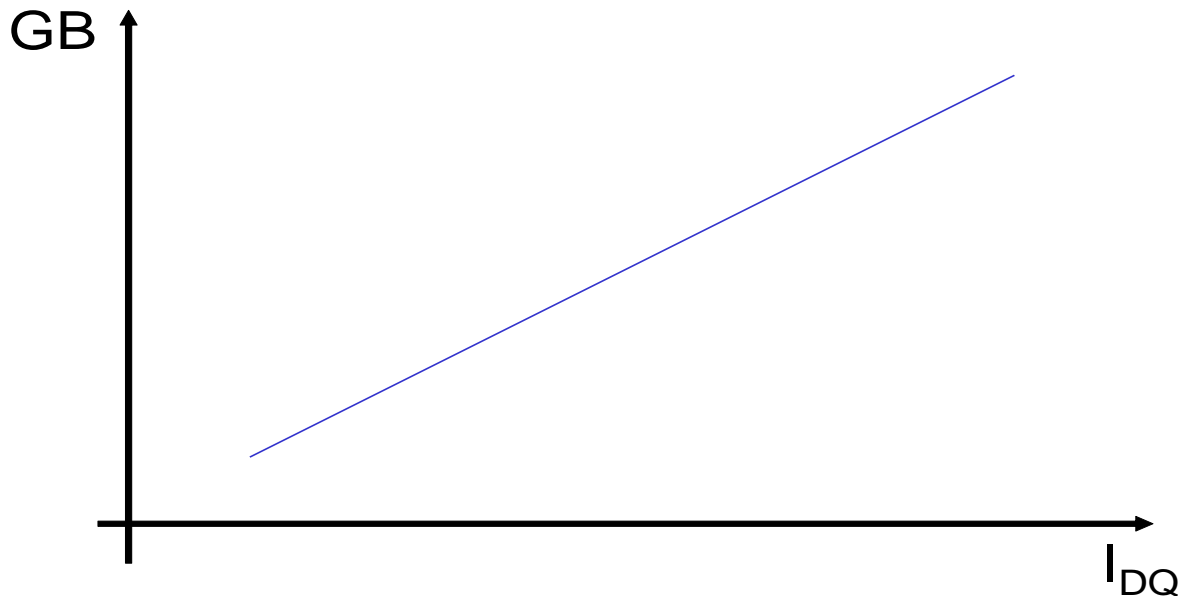
Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \left[\frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \left[\sqrt{\frac{W}{L}} \sqrt{I_{DQ}} \right]$$

GB increases with the square root of I_{DQ}
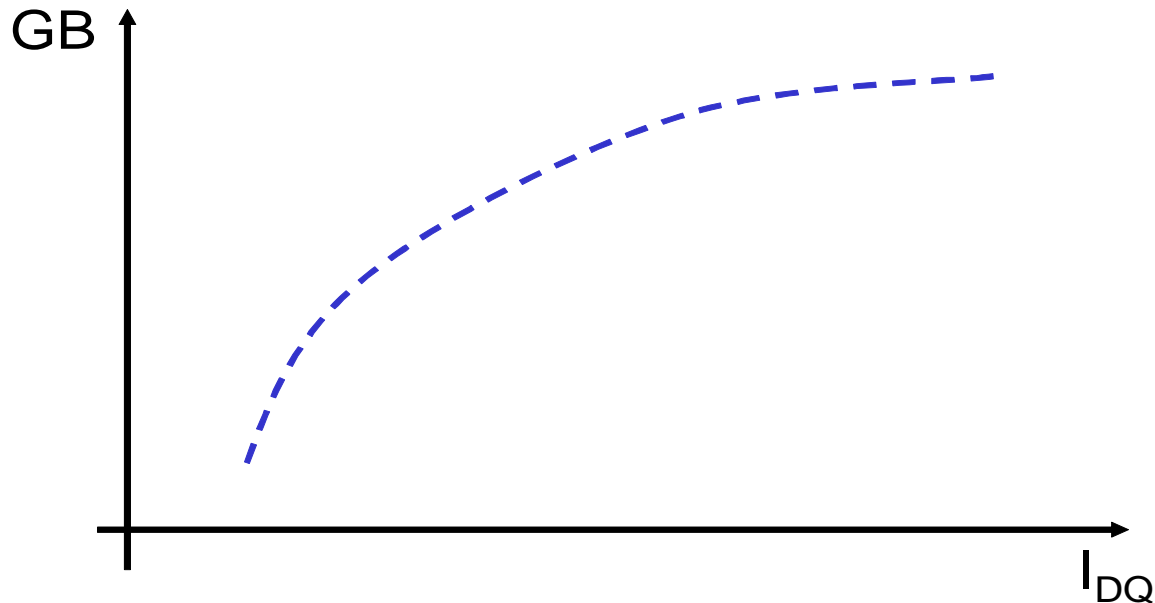
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Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \left[\frac{2}{V_{DD}C_L} \right] \left[\frac{P}{V_{EB}} \right]$$

GB independent of I_{DQ}
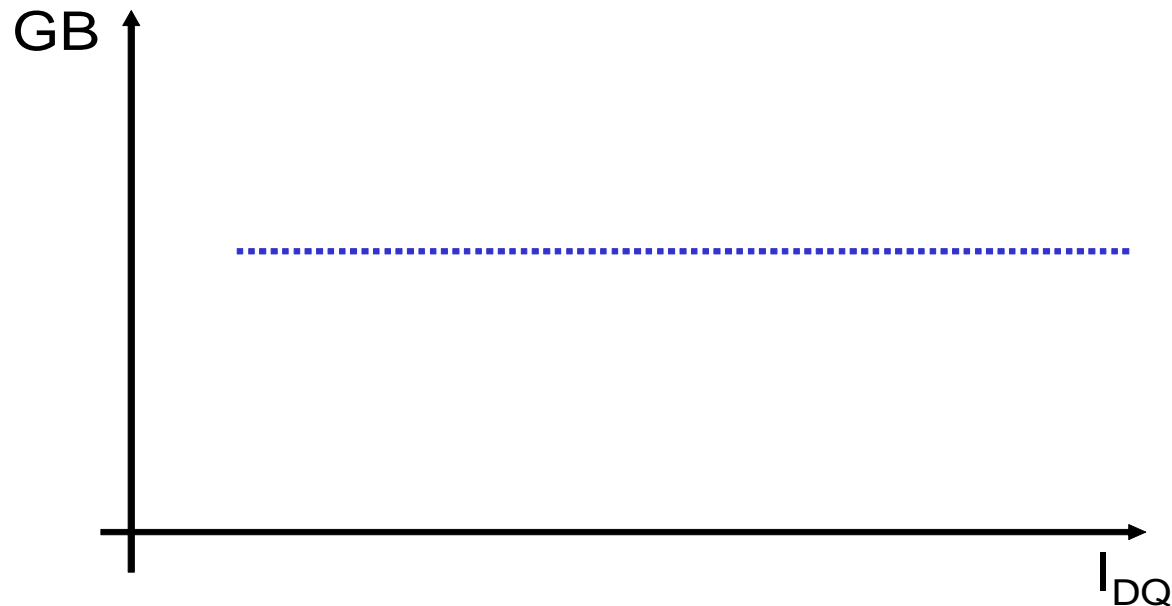
Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \frac{1}{\sqrt{I_{DQ}}} \frac{P}{C_L} \sqrt{\frac{2\mu C_{OX} W}{L}}$$

GB decreases with the reciprocal of the square root of I_{DQ}
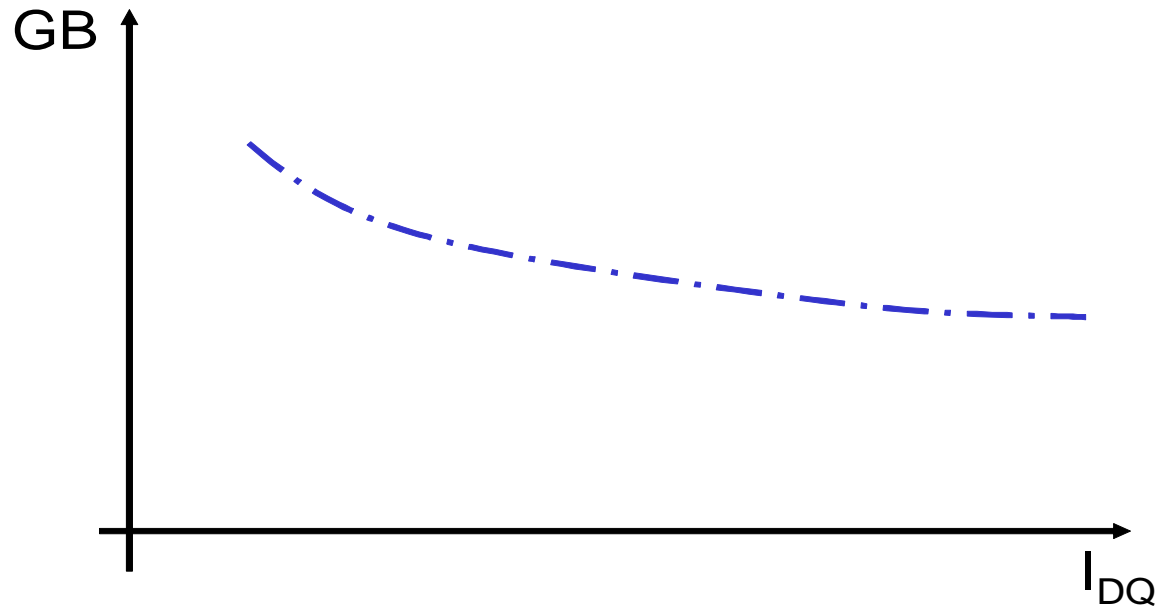
16

Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \frac{\sqrt{\dfrac{2\mu C_{OX} W P^3}{L V_{DD}}}}{I_{DQ} C_L}$$

GB decreases with the reciprocal of I_{DQ}

Design Space Exploration

Question: How does the GB of the single-stage amplifier change with bias current?

$$GB = \left[\frac{2}{C_L}\right]\left[\frac{I_{DQ}}{V_{EB}}\right] \quad \Longrightarrow \quad \text{Increases Linearly}$$

$$GB = \left[\frac{\sqrt{2\mu C_{OX}}}{C_L}\right]\left[\sqrt{\frac{W}{L}}\sqrt{I_{DQ}}\right] \quad \Longrightarrow \quad \text{Increases Quadratically}$$

$$GB = \left[\frac{2}{V_{DD}C_L}\right]\left[\frac{P}{V_{EB}}\right] \quad \Longrightarrow \quad \text{Independent of } I_{DQ}$$

$$GB = \frac{1}{\sqrt{I_{DQ}}}\frac{P}{C_L}\sqrt{\frac{2\mu C_{OX}W}{L}} \quad \Longrightarrow \quad \text{Decreases Quadradicly}$$

$$GB = \frac{\sqrt{\dfrac{2\mu C_{OX}WP^3}{LV_{DD}}}}{I_{DQ}C_L} \quad \Longrightarrow \quad \text{Decreases Linearly}$$

It depends upon how the design space is explored !!!

Design Space Exploration
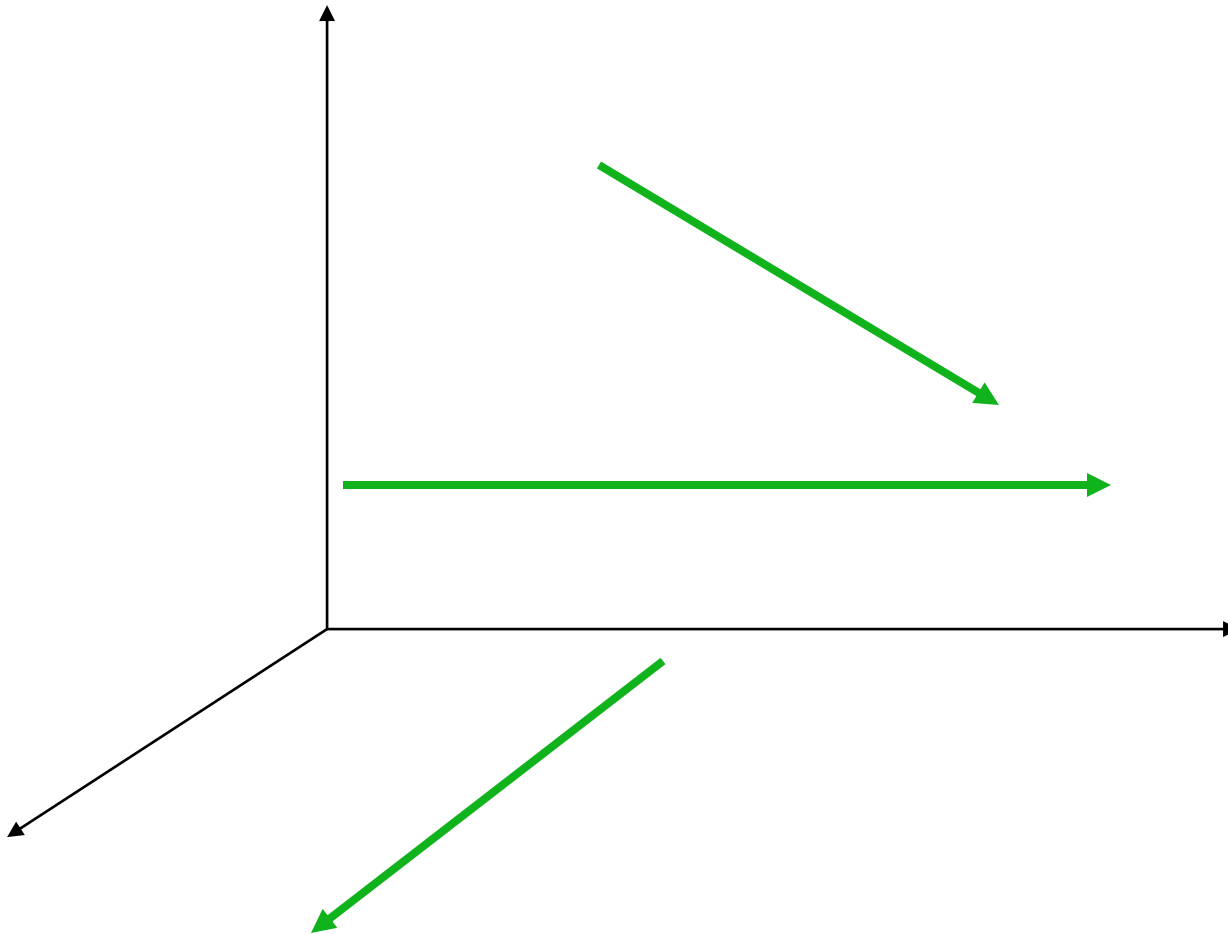
Different trajectories through a design space

Design Space Exploration

Issue becomes more involved for amplifiers or circuits with more than one transistor

Choice of design parameters can have major impact on insight into design

Size of parameter domain should agree with the number of degrees of freedom

Affects of any parameter on performance whether it be in the identified parameter domain or not is strongly dependent on how design space is explored

Small signal and natural parameter domains give little insight into design or performance

Single-Stage Low-Gain Op Amps

- Single-ended input

A_V

V_{DD}

I_{DQ}

V_{OUT}

C_L

M_1

V_{in}

V_{XX}

V_{SS}
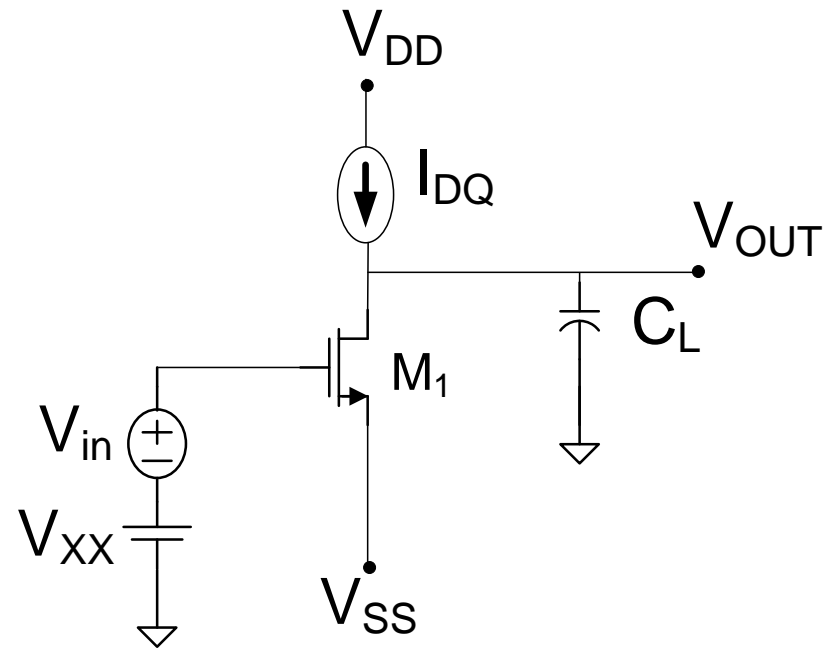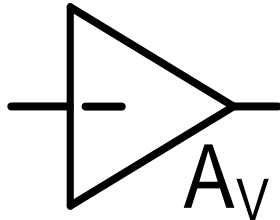
Basic single-stage op amp

Single-Stage Low-Gain Op Amps

- Single-ended input

A_V
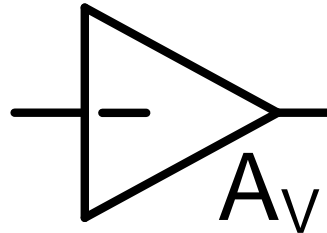
Observations:

- **This circuit often known as a common source amplifier**
- **Gain in the 30dB to 45dB range**
- **Inherently a transconductance amplifier since output impedance is high**
- **Voltage gain is ratio of transconductance gain to output conductance**
- **Critical to know degrees of freedom in design and know how to systematically explore design space**
- **Alternative parameter domain much more useful for design than small-signal domain or natural domain**
- **Performance of differential circuits will be obtained by inspection from those of the single-ended structures**

Review

- Multiple parameter domains can be used to characterize and explore a design space
- Performance characteristics of interest take on many different forms depending upon how design space is characterized
- Critical to identify the real number of degrees of freedom in design space (mathematical degrees of freedom minus the number of constraints)
- Performance characteristics often can be expressed as product of a process dependent term and an architecture dependent term
  - Facilitates comparison of different architectures
- Choice of characterization parameters can make a major difference on how hard it is to explore a design space

Review

- Design space is often a high-dimensional system with many local extrema (minimums or maximums)

- Be careful about drawing conclusions about how any parameter individually affects system performance because its affect will depend upon how the design space is explored

Design Space for Single-Stage Op Amp

$$GB = \left[\frac{2}{V_{DD}C_L} \right] \left[\frac{P}{V_{EB}} \right]$$
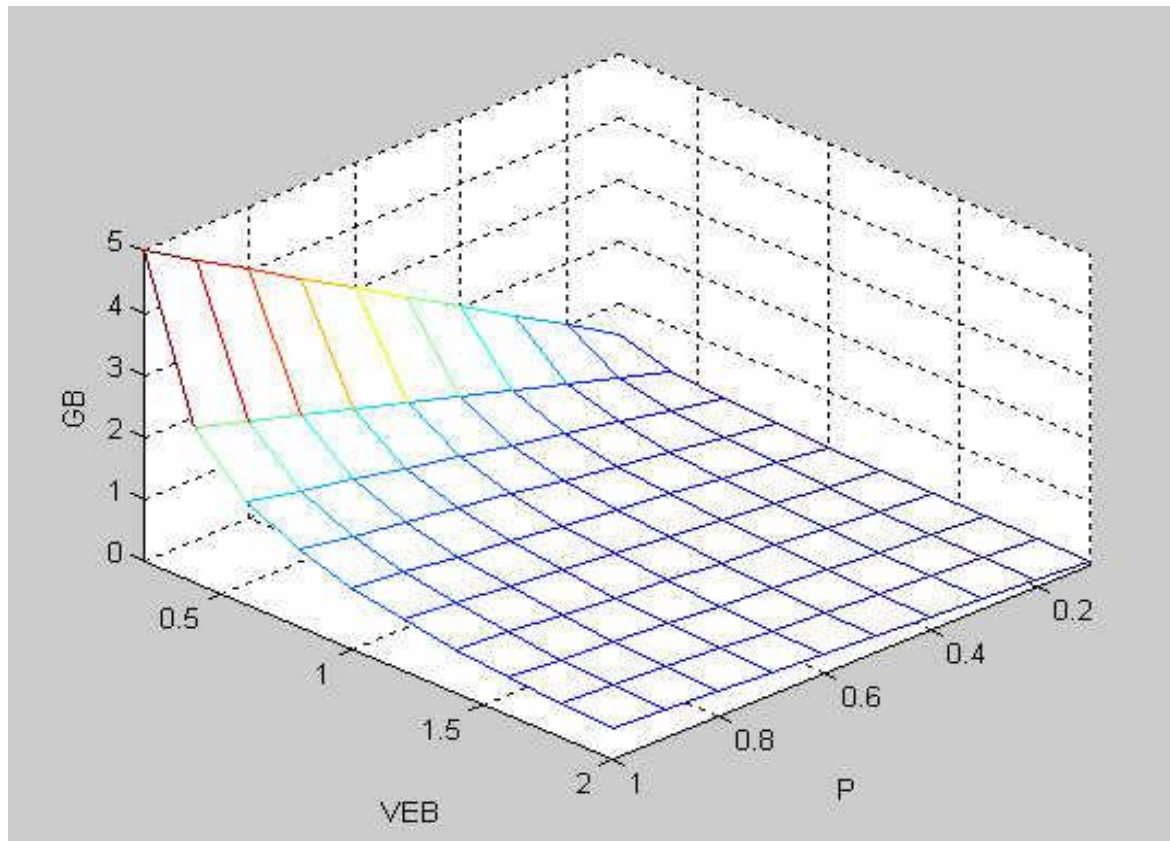
Plot of $\quad GB_N = \dfrac{P}{V_{EB}}$

Can we say that GB increases linearly with P ?

Design Space for Single-Stage Op Amp

$$GB = \left[\frac{2}{V_{DD}C_L} \right] \left[\frac{P}{V_{EB}} \right]$$

Plot of $\quad GB_N = \dfrac{P}{V_{EB}}$
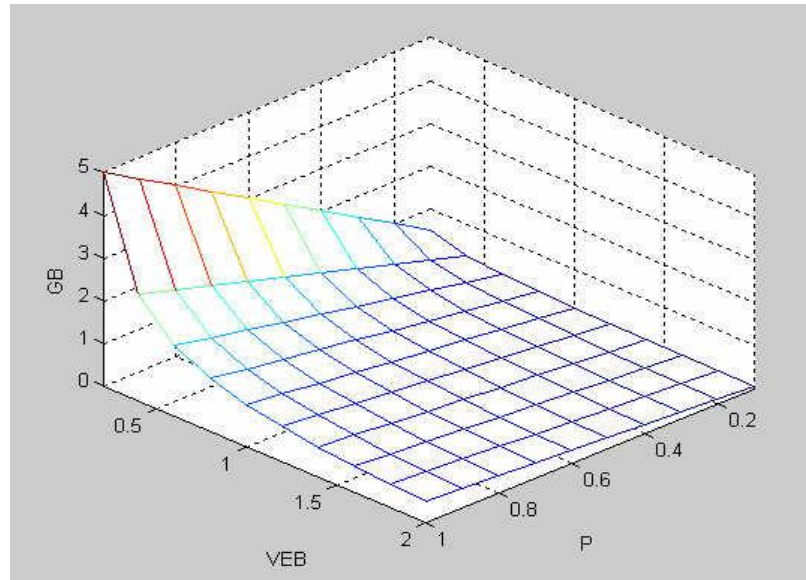
Can we say that GB increases linearly with P ?

Can we say that GB increases linearly with P if A_V is fixed?

Basic Op Amp Design

- Fundamental Amplifier Design Issues

→ - Single-Stage Low Gain Op Amps

- Single-Stage High Gain Op Amps

- Two-Stage Op Amp

- Other Basic Gain Enhancement Approaches

Single-Stage Low-Gain Op Amps
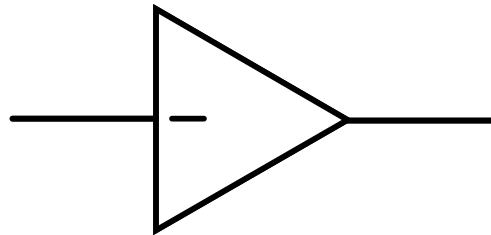
• Single-ended input

• Differential Input
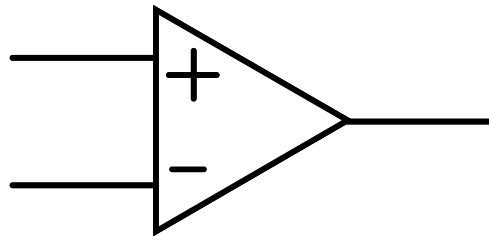
(Symbol does not distinguish between different amplifier types)

Differential Input Low Gain Op Amps

Will Next Show That :

- Differential input op amps can be readily obtained from single-ended op amps

- Performance characteristics of differential op amps can be directly determined from those of the single-ended counterparts

Systematic strategies for designing and analyzing op amps

- Analytical expressions for even simple op amps can become very complicated if brute force analysis techniques are used

- Considerable insight into both performance and design can be obtained from a systematic strategy for design and analysis of op amps

- Most authors present operational amplifiers from an "appear and analyze" approach

A systematic strategy for designing and analyzing op amps will now be developed

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Symmetric Networks

Theorem: If a linear network is symmetric, then for all differential symmetric excitations, the small signal voltage is zero at all points on the axis of symmetry.

$$\frac{v_d}{2}$$

$$-\frac{v_d}{2}$$

$v_X = 0$

Axis of
Symmetry
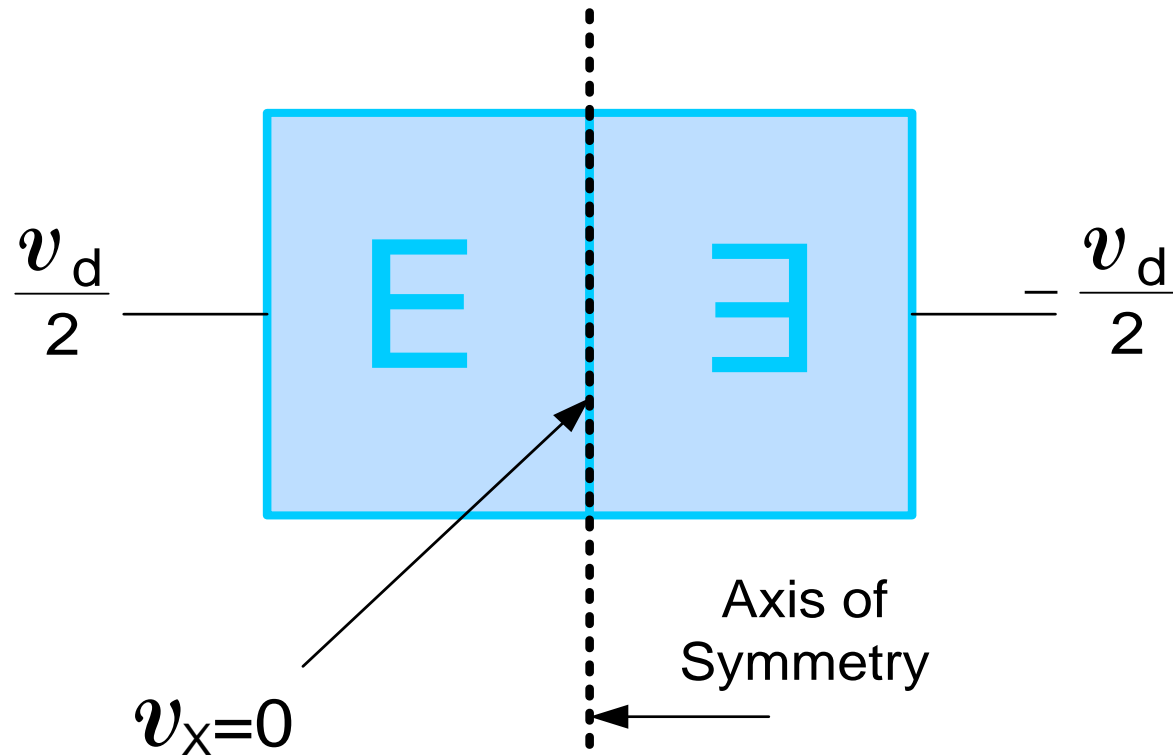
Counterpart Networks

Definition: The counterpart network of a network is obtained by replacing all n-channel devices with p- channel devices, replacing all p-channel devices with n-channel devices, replacing V_{SS} biases with V_{DD} biases, and replacing all V_{DD} biases with V_{SS} biases.

Counterpart Networks

Example:

Counterpart Networks

V_{DD}

M_2

M_1
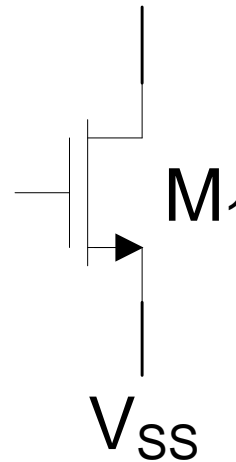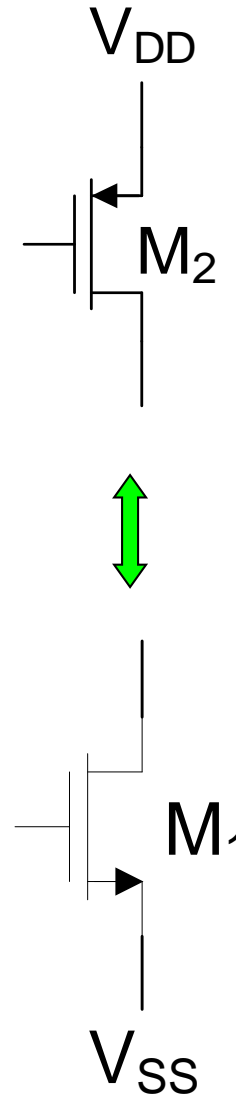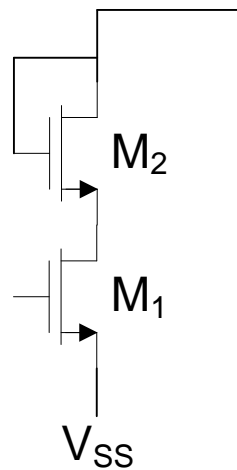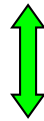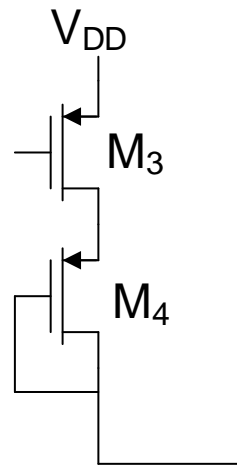
V_{SS}
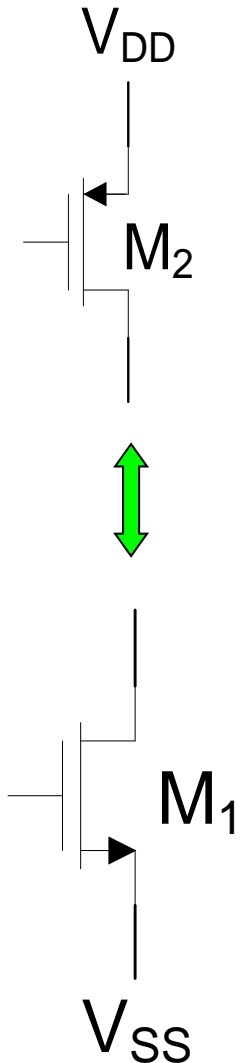
the counterpart network is unique

the counterpart of the counterpart is the original network

Counterpart Networks

Theorem: The parametric expressions for all small-signal characteristics[1], such as voltage gain, output impedance, and transconductance of a network and its counterpart network are the same.

[1]when expressed in the small-signal parameter domain

Terminology and Notation

F

Quarter Circuit

P

Counterpart Circuit

P

F

Half Circuit

q

Ⅎ

"Reflected Image" Half Circuit

Axis of Symmetry

P **q**

F **Ⅎ**

Symmetric Circuit

Reflected Image Half-circuit is identical to half-circuit, just drawn differently

Synthesis of fully-differential op amps from symmetric networks and counterpart networks

Theorem: If F is any network with a single input and P is its counterpart network, then the following circuits are fully differential circuits --- "op amps".
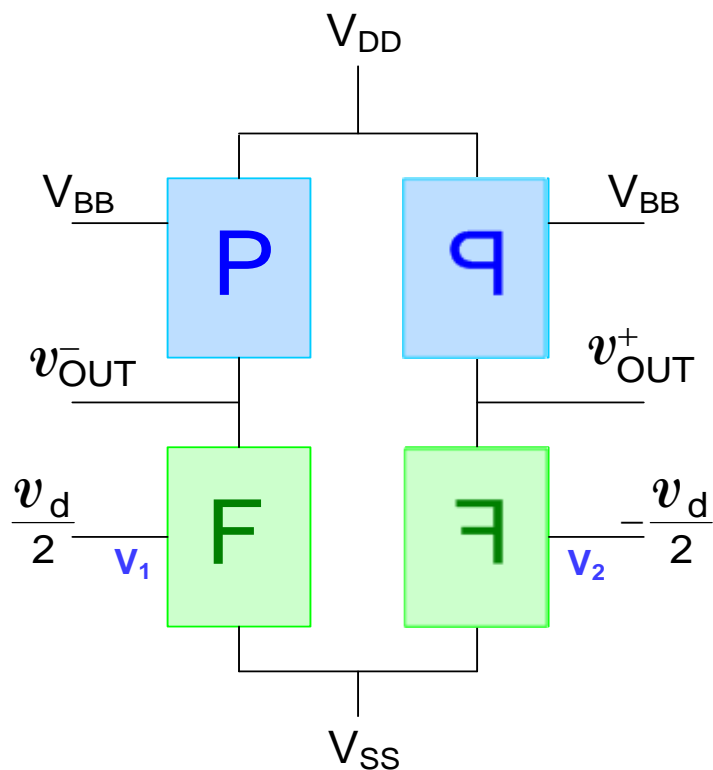
$$V_d = V_1 - V_2$$
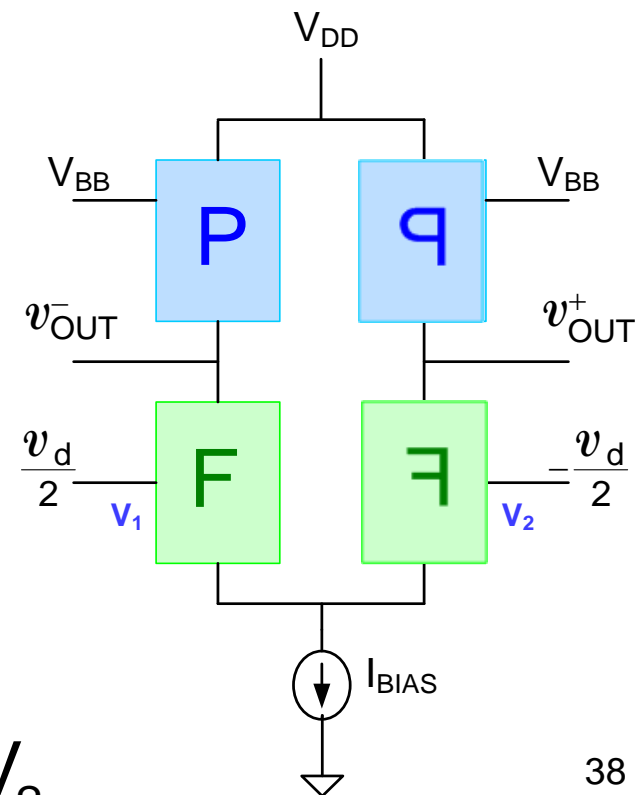
Synthesis of fully-differential op amps from symmetric networks and counterpart networks
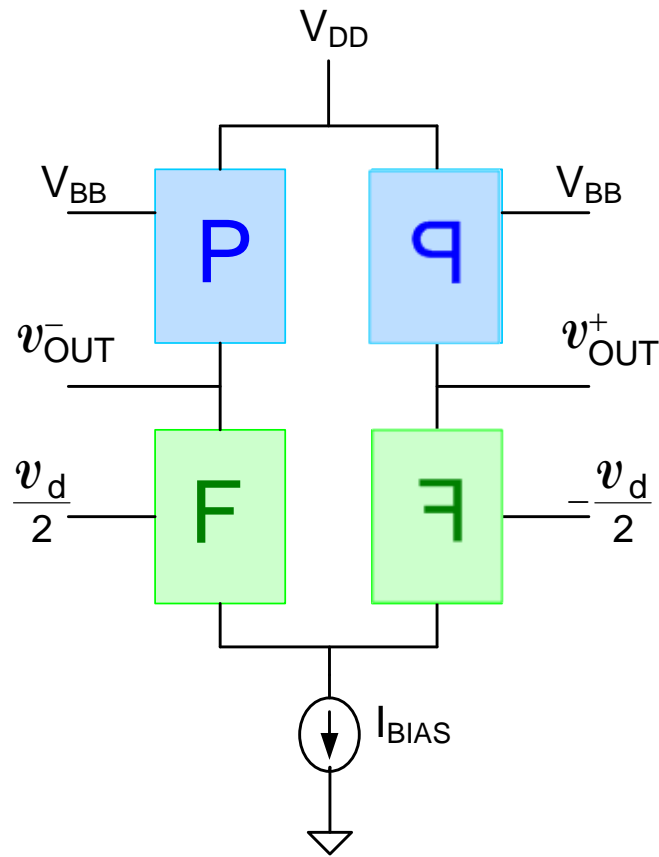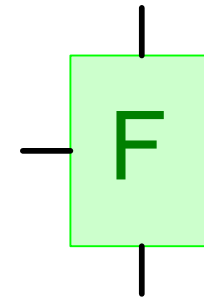
Quarter Circuit
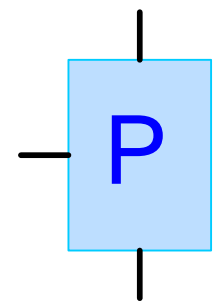
Counterpart Circuit
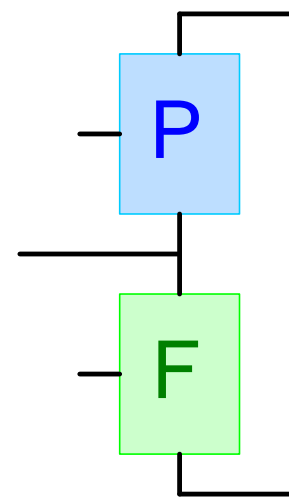
Half Circuit
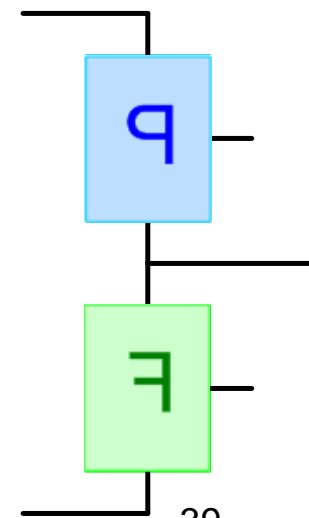
Reflected Image Half Circuit

$$\mathcal{v}_d = V_1 - V_2$$

Synthesis of fully-differential op amps from symmetric networks and counterpart networks
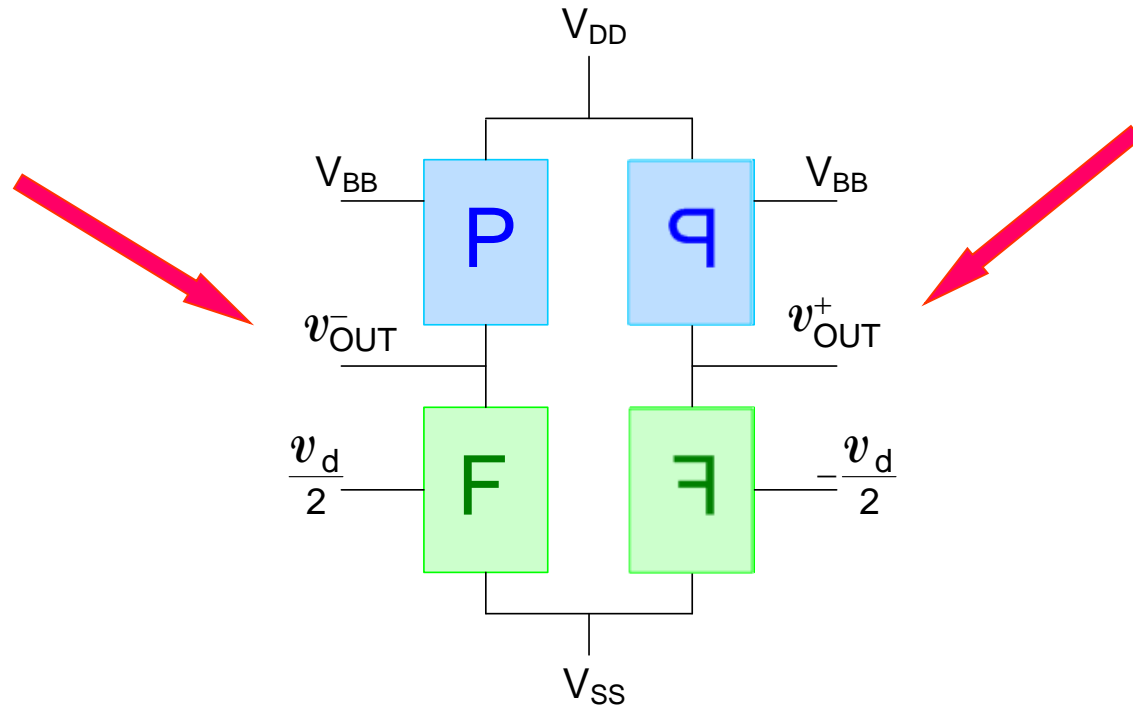
What do we do with the extra output ?
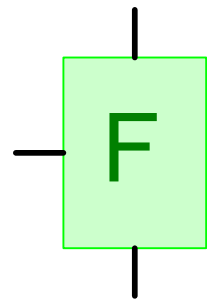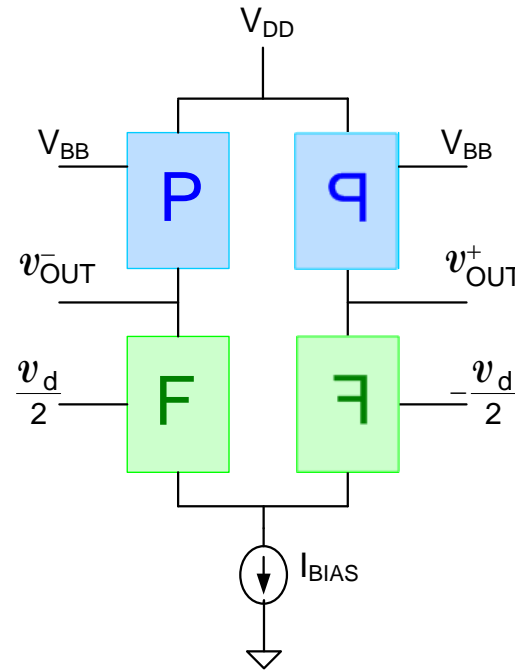
What do we do with the extra output ?

V_{DD}

V_{BB} P   Ꝗ V_{BB}

v_{OUT}^{-} v_{OUT}^{+}

$\dfrac{v_d}{2}$ F   Ⅎ $-\dfrac{v_d}{2}$

V_{SS}
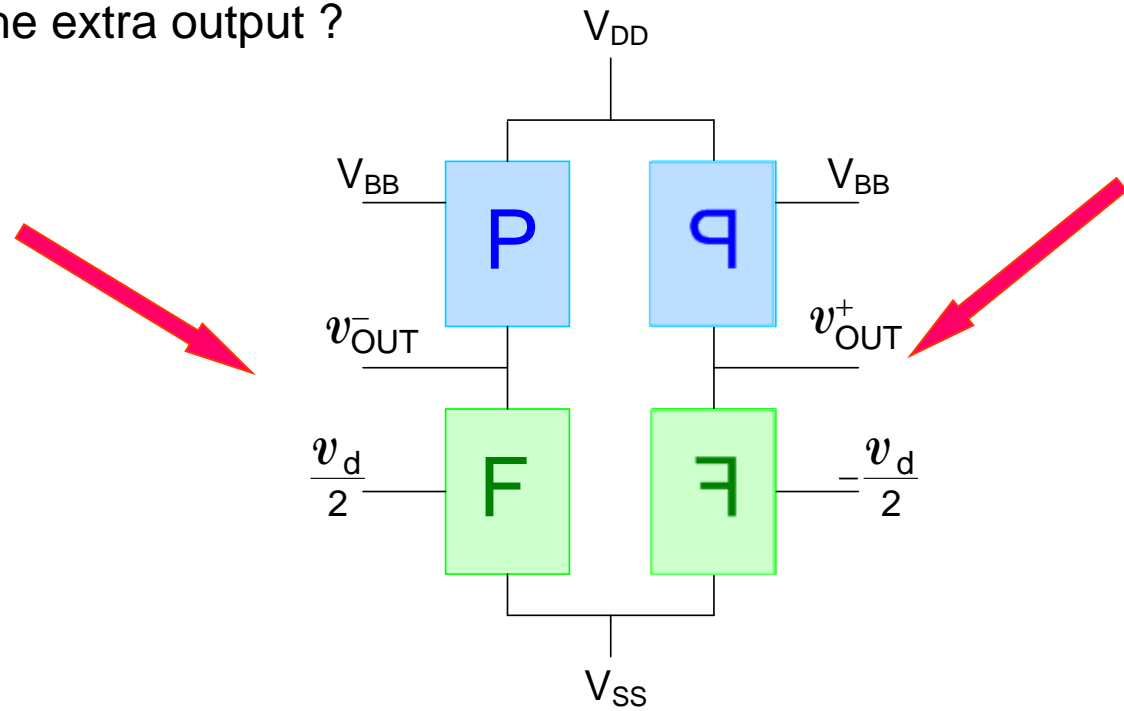
Use it or ignore it !!

Synthesis of fully-differential op amps from symmetric networks and counterpart networks

A fully differential op amp can be derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit with a current source on the axis of symmetry.
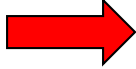

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

Synthesis of fully-differential op amps from symmetric networks and counterpart networks

A fully differential op amp can be derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit with a current source on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

Implications: Much Op Amp design can be reduced to designing much simpler quarter-circuits where it is much easier to get insight into circuit performance
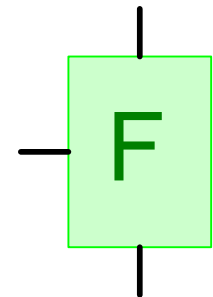
F

Quarter Circuit

- **General Differential Analysis**
- 5T Op Amp from simple quarter circuit
- Biasing with CMFB circuit
- Common-mode and differential-mode analysis
- Common Mode Gain
- Overall Transfer Characteristics

Characterization of Quarter Circuit

If the input impedance is infinite and circuit is unilateral, the two-port network only has two characterizing parameters : G_M and G

Two-Port Model of Quarter Circuit
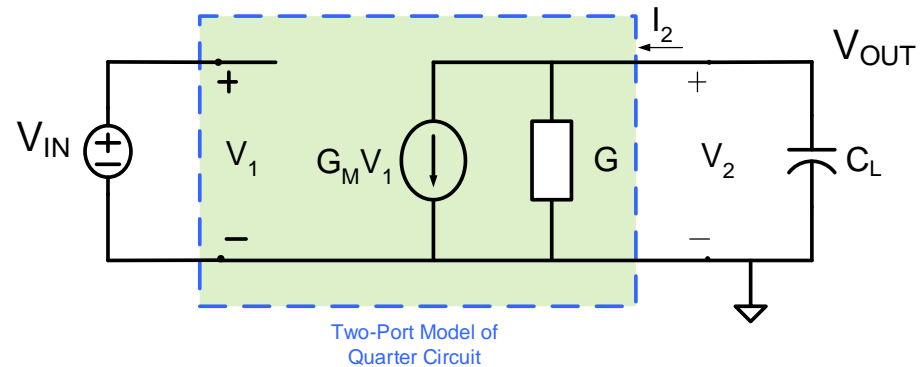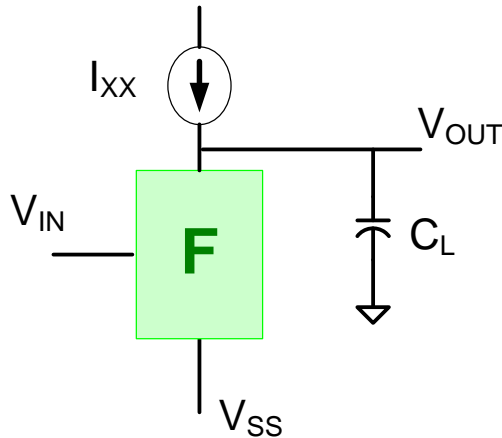
$$V_{OUT}(G + sC_L) + G_M V_1 = 0$$
$$V_{IN} = V_1$$

$$\Longrightarrow \quad A_{VQC}(s) = \frac{-G_M}{sC_L + G}$$
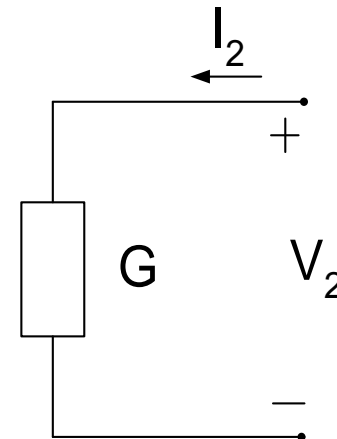
$$A_{VOQC} = -\frac{G_M}{G}$$

$$BW = \frac{G}{C_L}$$

$$GB = \frac{G_M}{C_L}$$

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Characterization of Quarter Circuit (or Counterpart Circuit) with input port terminated in short circuit
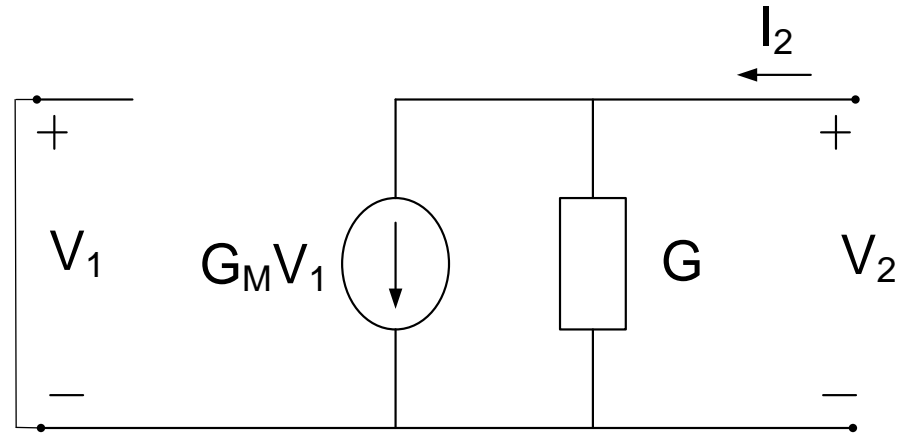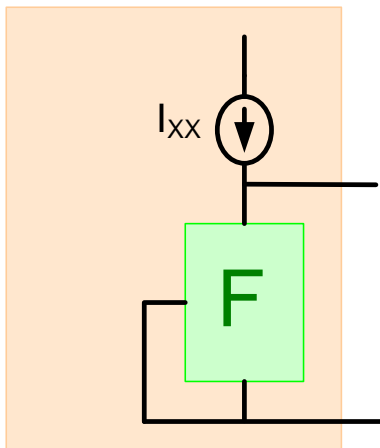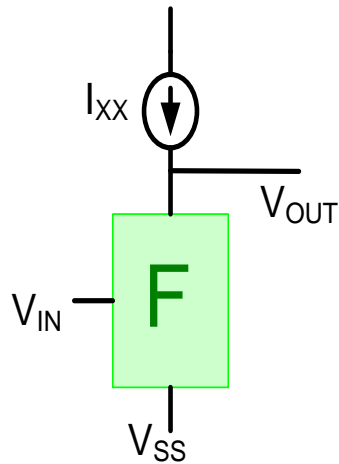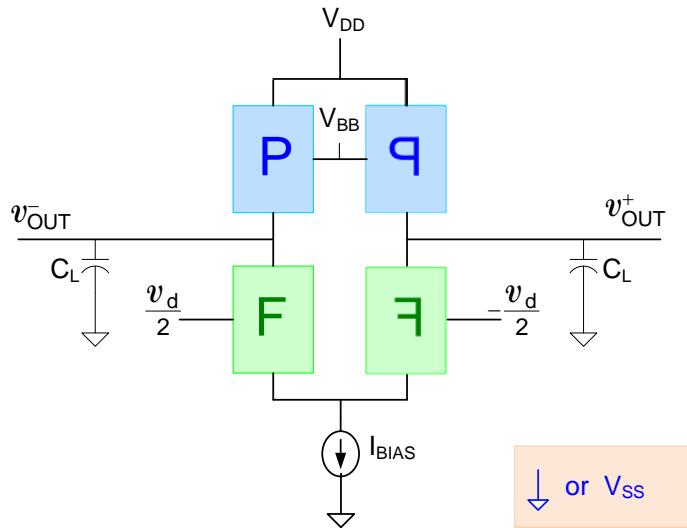

If the input port of a two-port has an ac short, then the two-port reduces to a one-port characterized by the conductance G
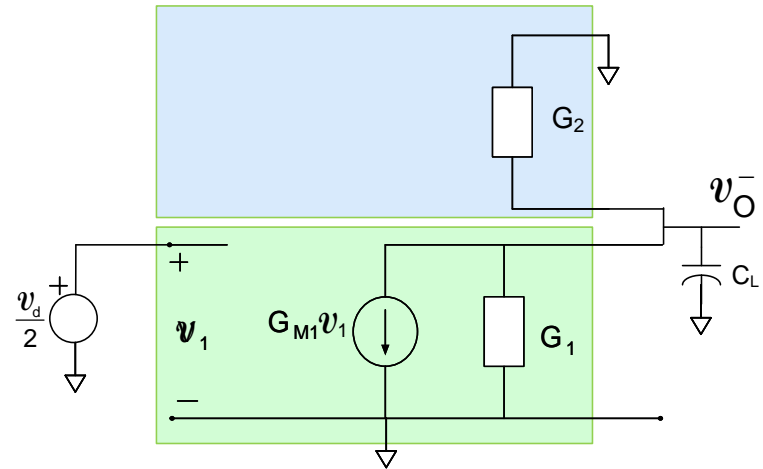
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Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal differential half-circuit
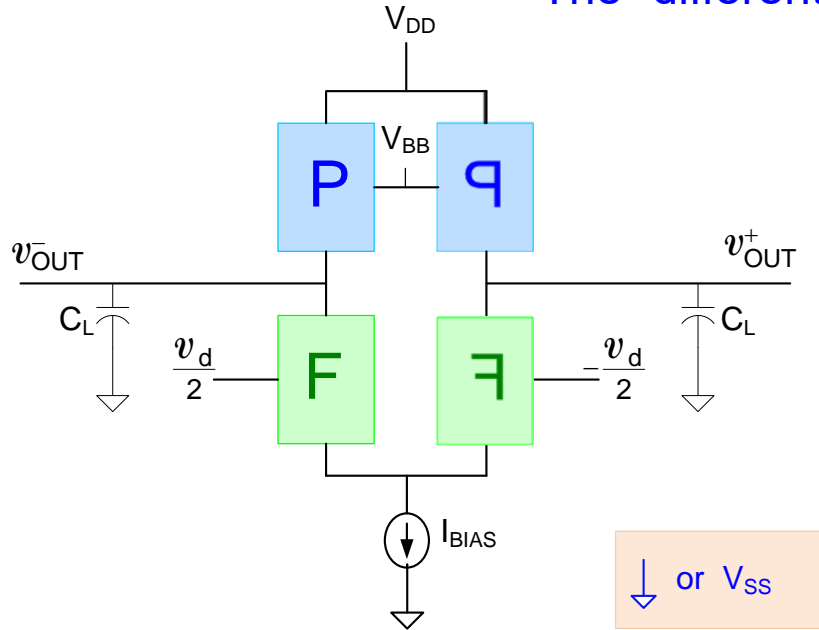

Derivation:

from KCL and KVL:

$$v_o^- (G_1 + G_2 + sC_L) + G_{M1}v_1 = 0$$

$$v_1 = \frac{v_d}{2}$$

$$\Bigg\} \implies$$

$$A_V = \frac{v_o^-}{v_d} = \frac{-\dfrac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

Note: Factor of 2 reduction of gain since only half of the differential input is applied to the half-circuit

Note: More reduction of gain since denominator increases

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Determination of op amp characteristics from quarter circuit characteristics
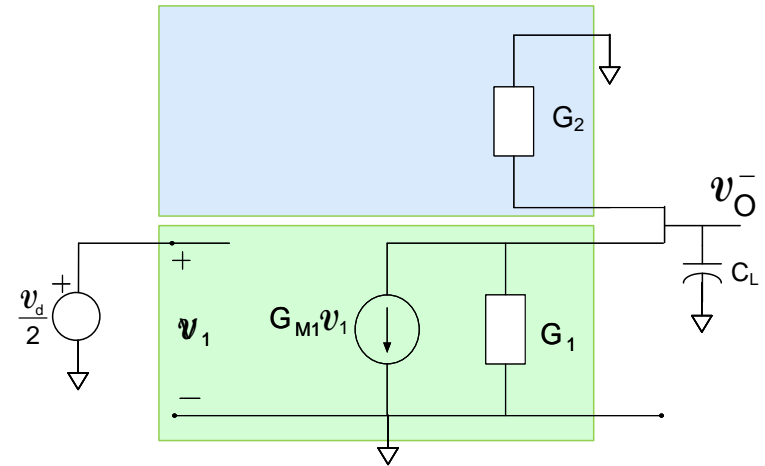
-- The "differential" gain --

Small signal differential half-circuit

$$A_V = \frac{v_O^-}{v_d} = \frac{-\dfrac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

$A_{V0}=?$
$BW=?$
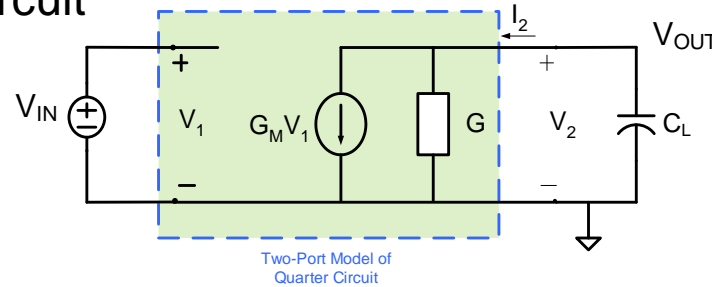$GB=?$

$$A_{VO} = \frac{-G_{M1}}{2(G_1 + G_2)}$$

$$BW = \frac{G_1 + G_2}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$

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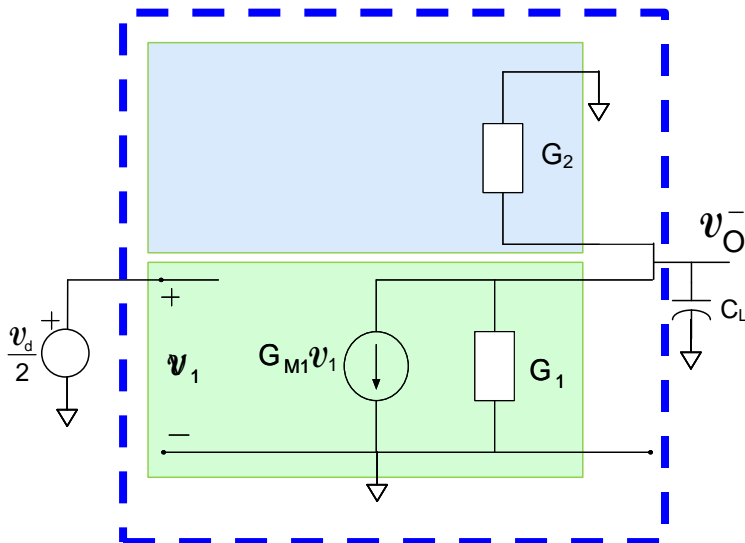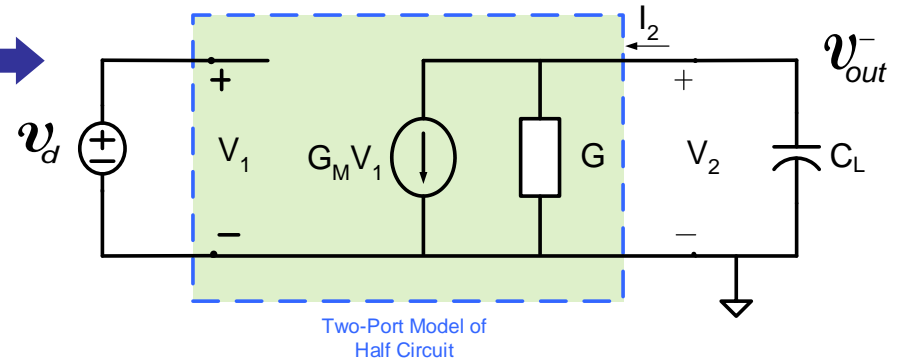# Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal Quarter Circuit

Two-Port Model of Quarter Circuit

$$A_{VQC}(s) = \frac{-G_M}{sC_L + G}$$

Small signal differential half-circuit (repeated from last slide)

$$G_M = \frac{G_{M1}}{2}$$

$$G = G_1 + G_2$$

Two-Port Model of Half Circuit

$$A_V = \frac{v_o^-}{v_d} = \frac{-\frac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$
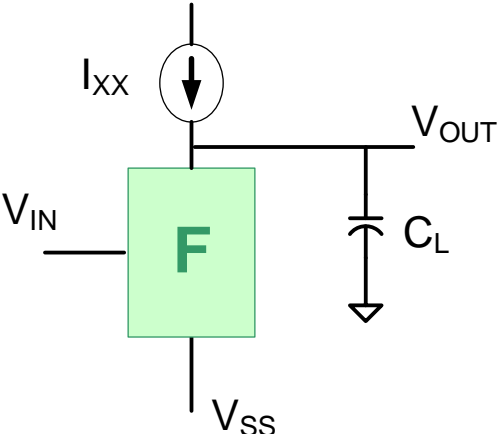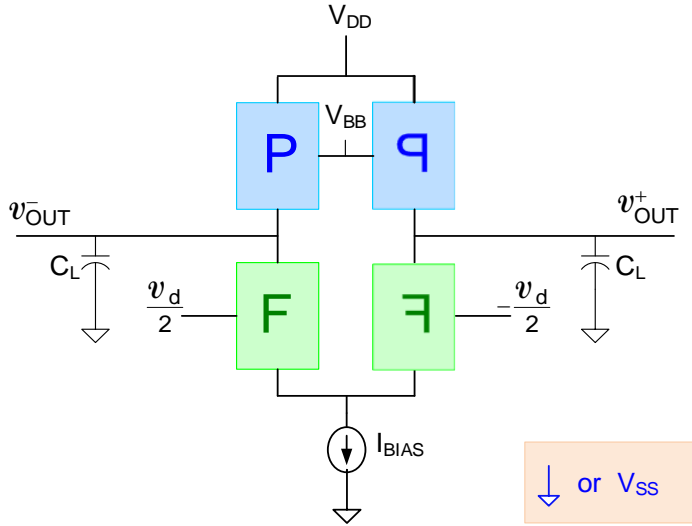
$$A_{VQC}(s) = \frac{-G_M}{sC_L + G}$$

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Determination of op amp characteristics from quarter circuit characteristics

-- The "differential" gain --

Small signal Quarter Circuit

Small signal differential amplifier
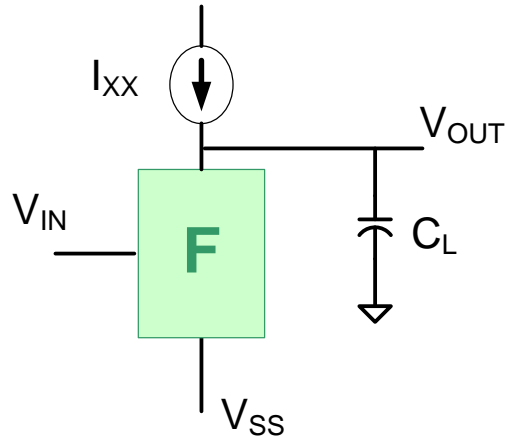

$$A_{VQC}(s) = \frac{-G_M}{sC_L + G}$$

$$\Longleftrightarrow$$

$$A_V = \frac{v_o^-}{v_d} = \frac{-\dfrac{G_{M1}}{2}}{sC_L + G_1 + G_2}$$

Determination of op amp characteristics from quarter circuit characteristics
-- The "differential" gain --

Small signal Quarter Circuit
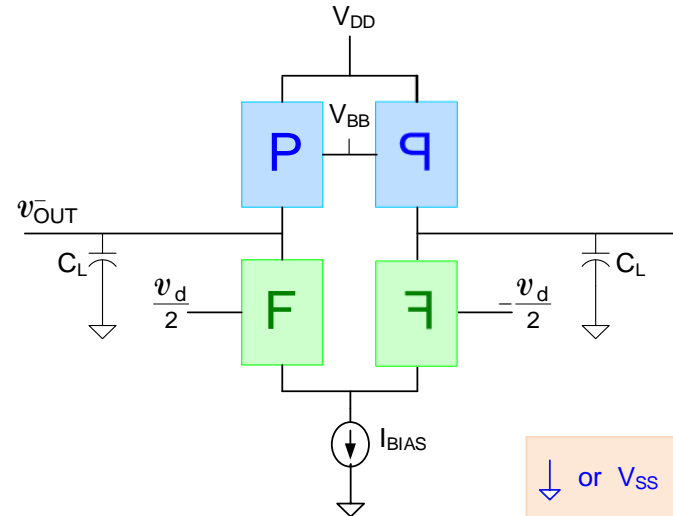

$$A_{VOQC} = -\frac{G_M}{G}$$

$$BW = \frac{G}{C_L} \qquad GB = \frac{G_M}{C_L}$$

$\longleftrightarrow$

Small signal differential amplifier

↓ or V_{SS}

$$A_{V0} = \frac{v_{OUT}^-}{v_d} = \frac{-G_{M1}}{2(G_1 + G_2)}$$
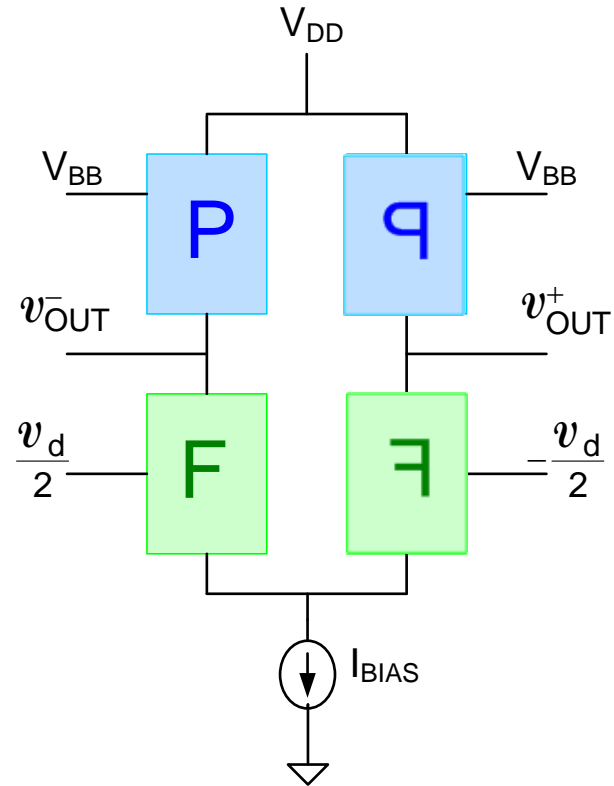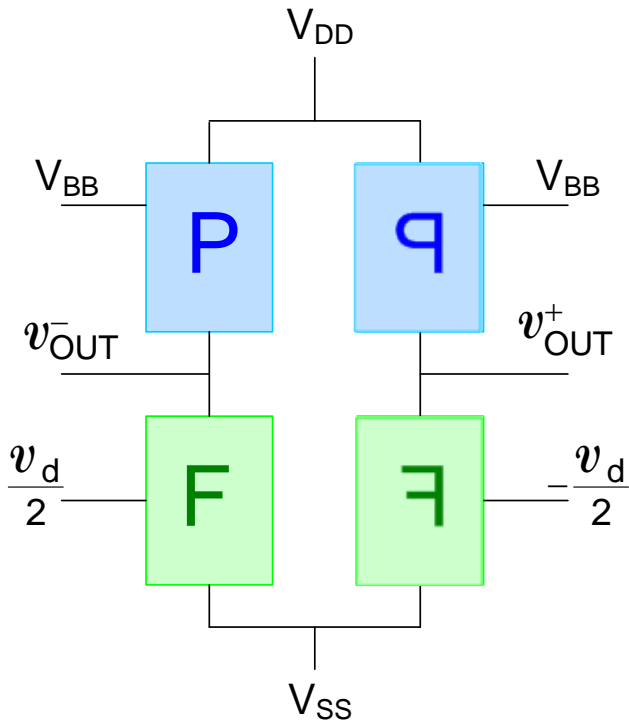
$$BW = \frac{G_1 + G_2}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$

Note: Factor of 4 reduction of gain if $G_1 = G_2$ (this often occurs)

Note: Factor of 2 increase of BW if $G_1 = G_2$ (this often occurs)

Note: Factor of 2 reduction of GB if $G_1 = G_2$ (this often occurs)

Remember this is applicable to ANY quarter circuit !

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# Comparison of Tail Voltage and Tail Current Source Structures

Small signal half-circuits are identical so differential voltage gains, BW, and GB are all the same
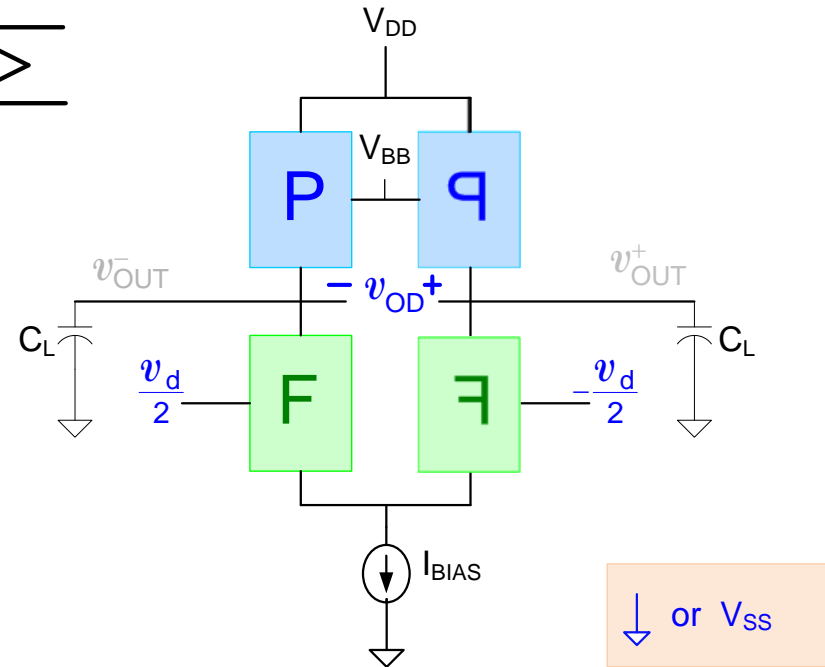
Biasing Issues for Differential Amplifier

- Tail voltage bias not suitable for large common-mode (CM) input range but does offer good output swing

- Tail current bias provides good CM input range but at the expense of a modest reduction in output signal swing

Differential Output Amplifiers

-- The "differential" gain --
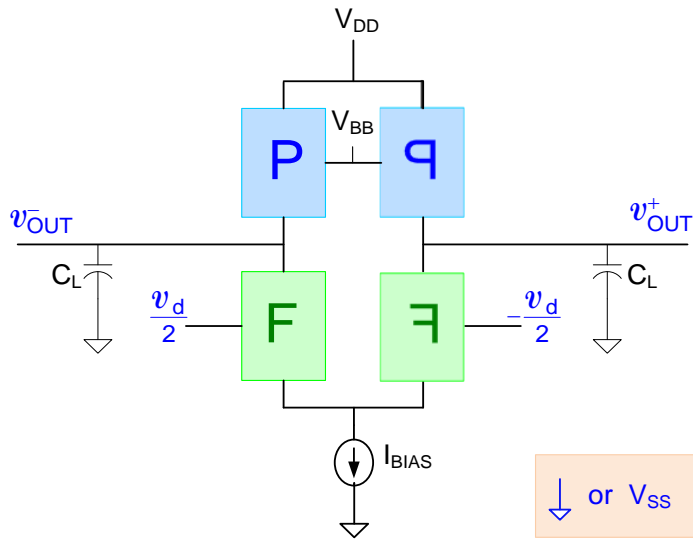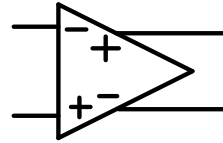
Single-Ended Outputs
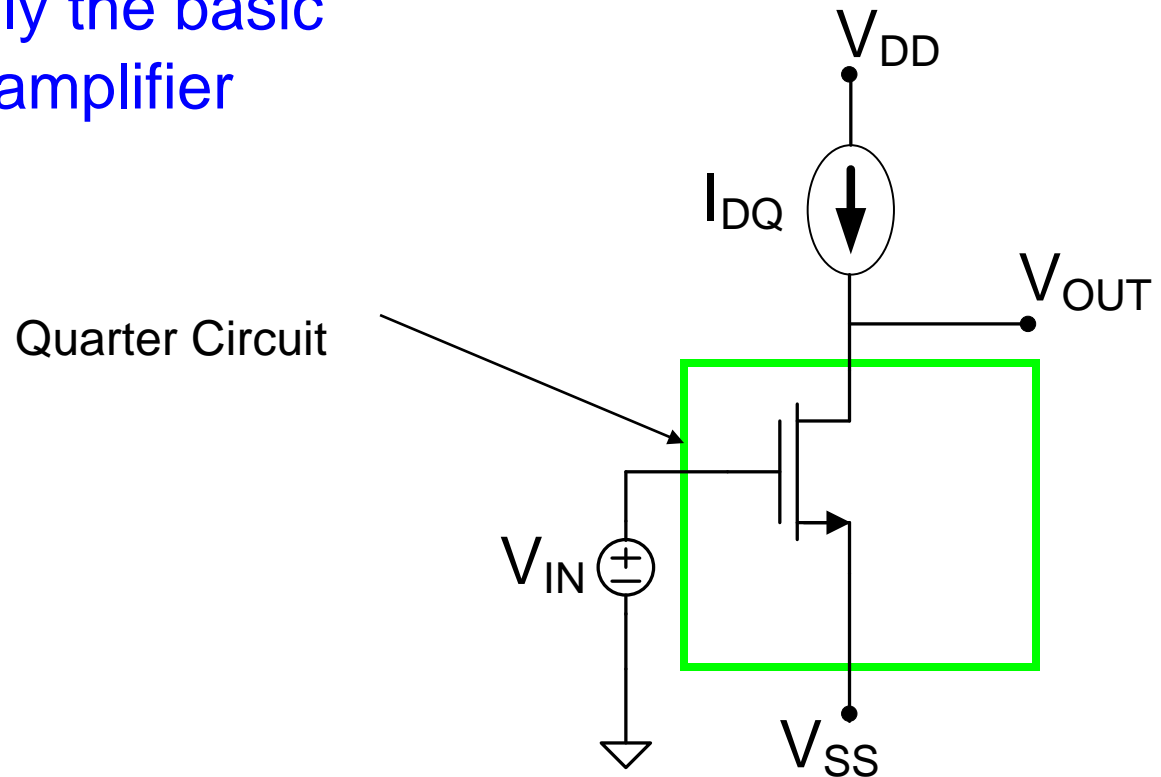
Differential Output

Theorem: For a symmetric circuit with symmetric outputs and differential excitations:

- Differential Voltage Gain Double that of Single-Ended Structure
- BW is the same
- GB Doubles for the Differential Output Structure

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- General Differential Analysis
- 5T Op Amp from simple quarter circuit
- Biasing with CMFB circuit
- Common-mode and differential-mode analysis
- Common Mode Gain
- Overall Transfer Characteristics

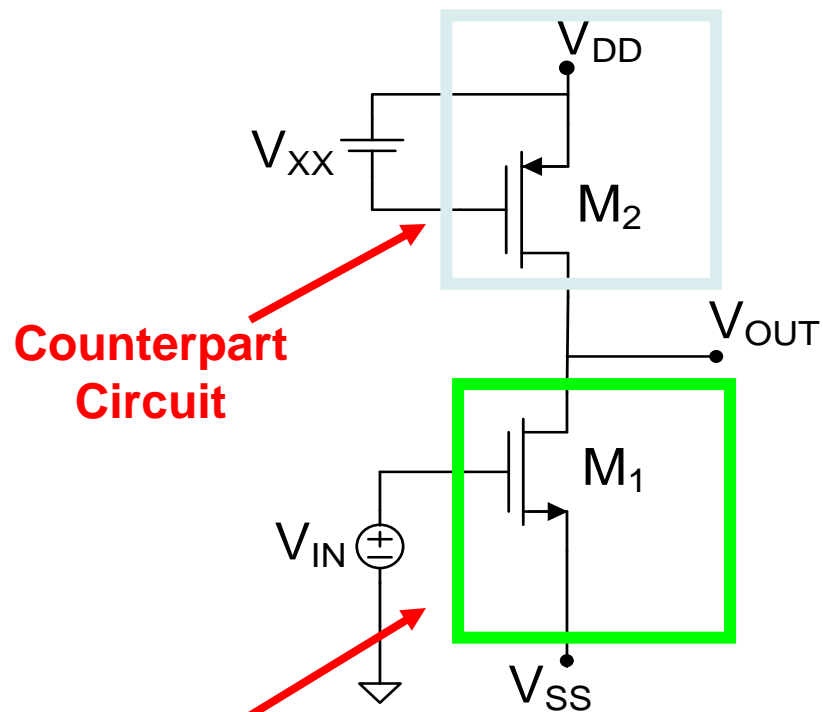# Applications of Quarter-Circuit Concept to Op Amp Design

consider initially the basic single-ended amplifier

Quarter Circuit

V_{DD}

I_{DQ}

V_{OUT}

V_{IN}

V_{SS}

Single-stage single-input low-gain op amp
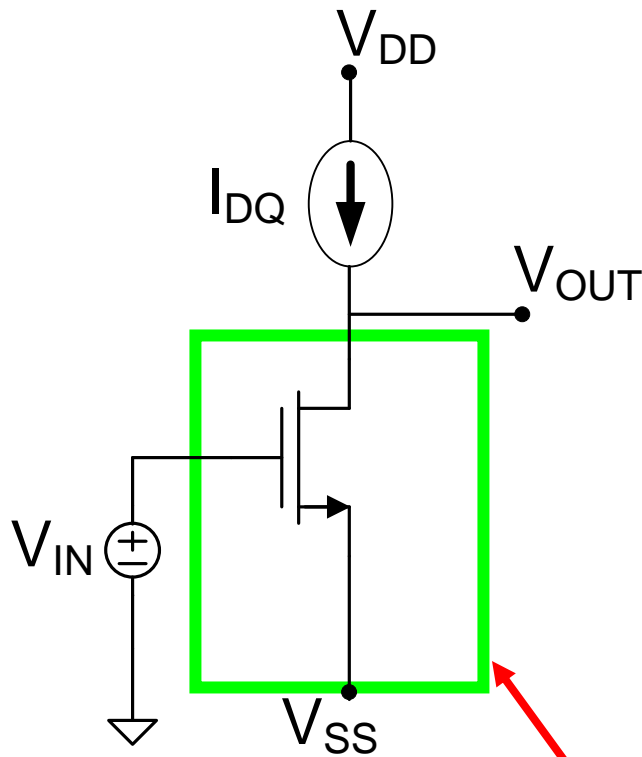
Basic Structure

Counterpart Circuit

Quarter Circuit

Practical Implementation

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Small signal model of half-circuit

Two-port model of half-circuit

$$G = G_1 + G_2$$

$$G_M = G_{M1}$$
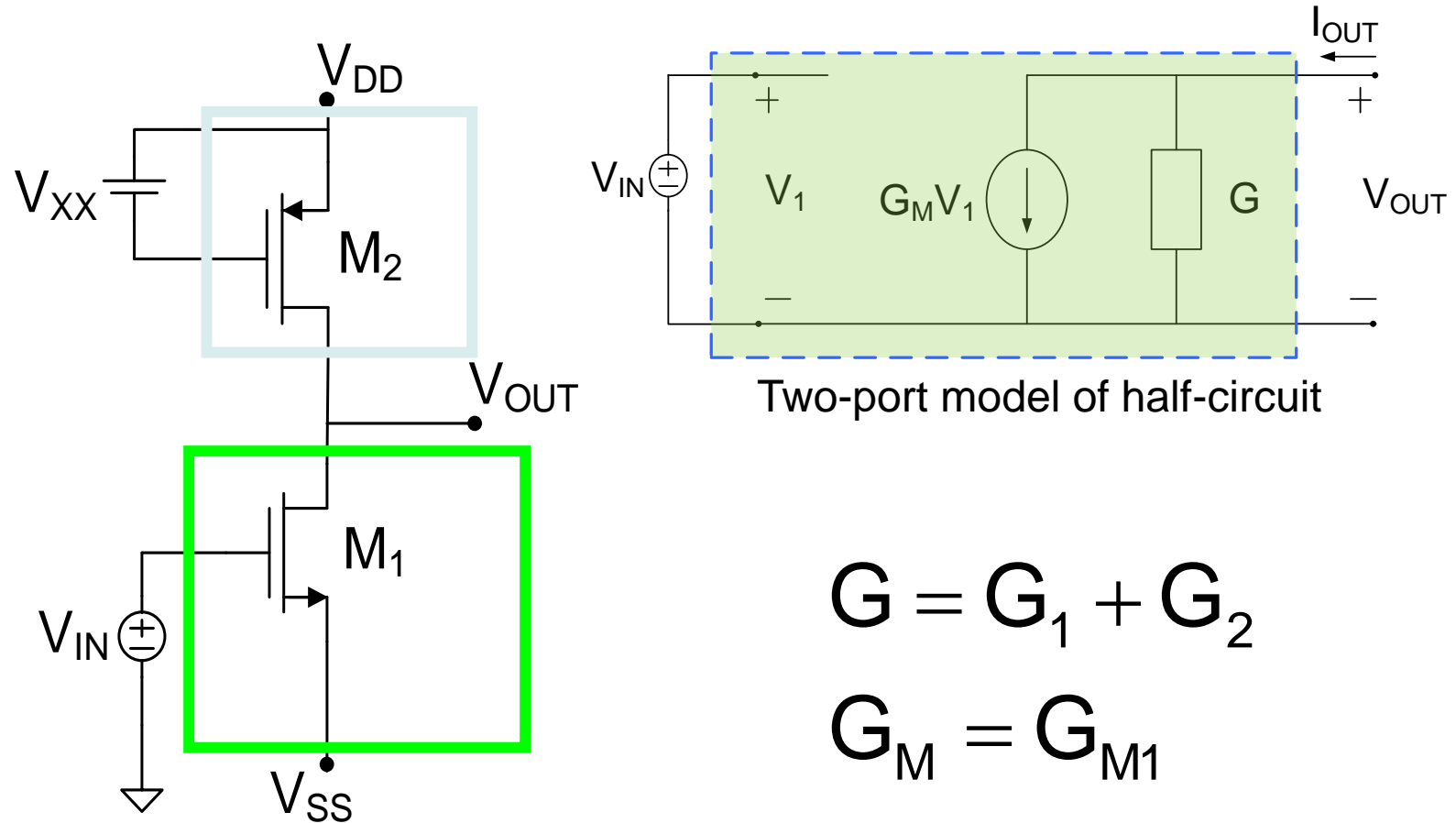
Single-stage low-gain differential op amp
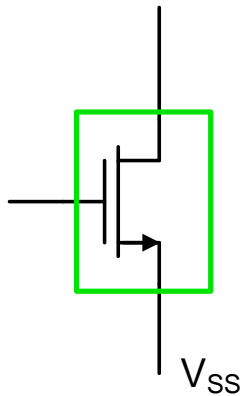
Quarter Circuit

Single-Ended Output : Differential Input Gain

$$A(s) = \frac{v_{OUT}}{v_d} = \frac{-\dfrac{g_{m1}}{2}}{sC_L + g_{o1} + g_{o3}}$$

$$A_{V0} = \frac{-g_{m1}}{2(g_{o1} + g_{o3})}$$

$$BW = \frac{g_{o1} + g_{o3}}{C_L}$$

$$GB = \frac{g_{m1}}{2C_L}$$

Circuit is Very Sensitive to V_{B1} and V_{B2} !!

- Have synthesized fully differential op amp from quarter circuit !
- Have obtained analysis of fully differential op amp directly from quarter circuit !
- Still need to determine what happens if input is not differential !

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- General Differential Analysis
- 5T Op Amp from simple quarter circuit
→ Biasing with CMFB circuit
- Common-mode and differential-mode analysis
- Common Mode Gain
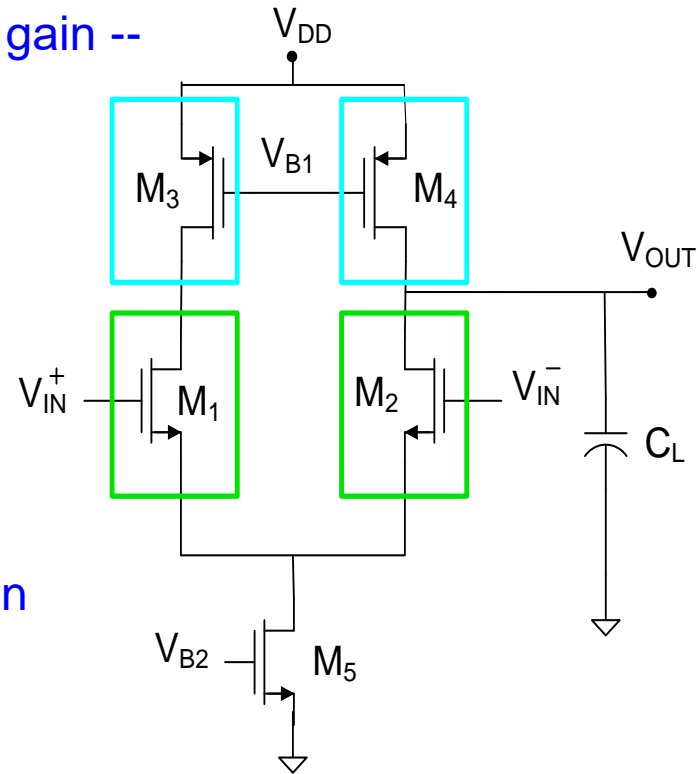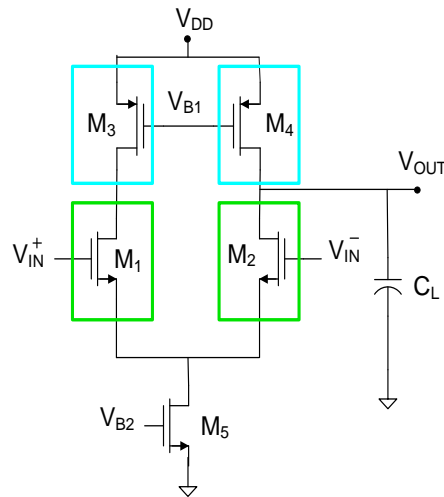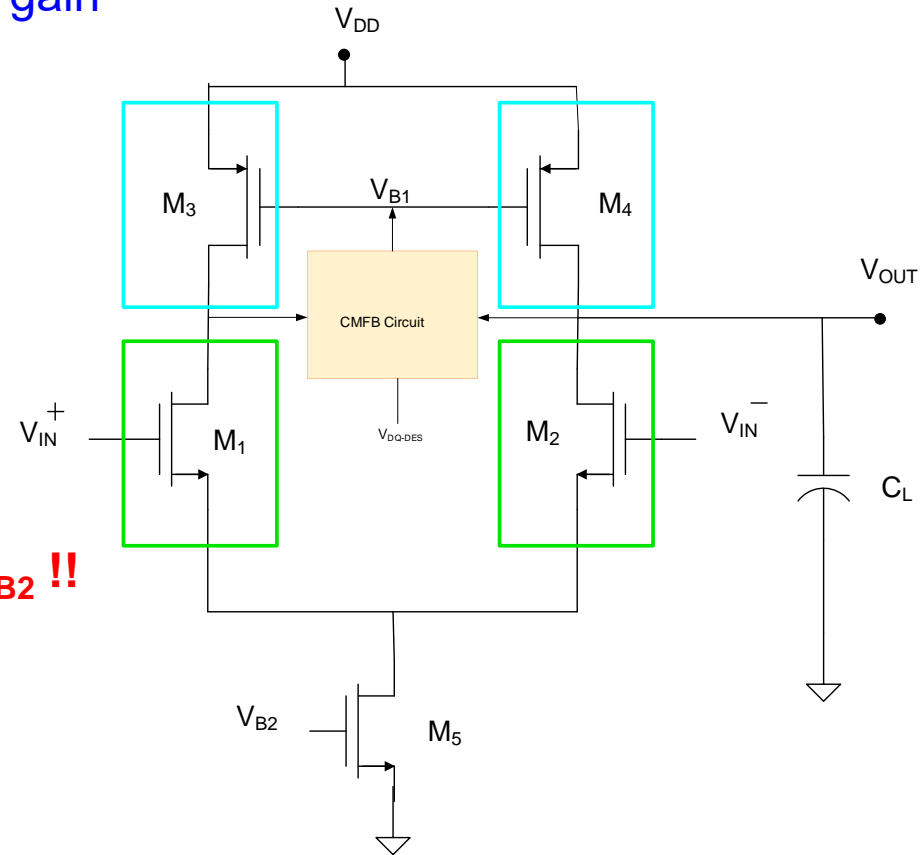- Overall Transfer Characteristics

Single-stage low-gain differential op amp

-- The "differential" gain --

**Need CMFB circuit to establish V_{B1} or V_{B2} !!**

- CMFB circuit determines average value of the drain voltages
- Compares the average to the desired quiescent drain voltages
- Established a feedback signal V_{B1} to set the right Q-point
- Shown for V_{B1} but could alternately be applied to V_{B2}

Details about CMFB circuits will be discussed later

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Single-stage low-gain differential op amp

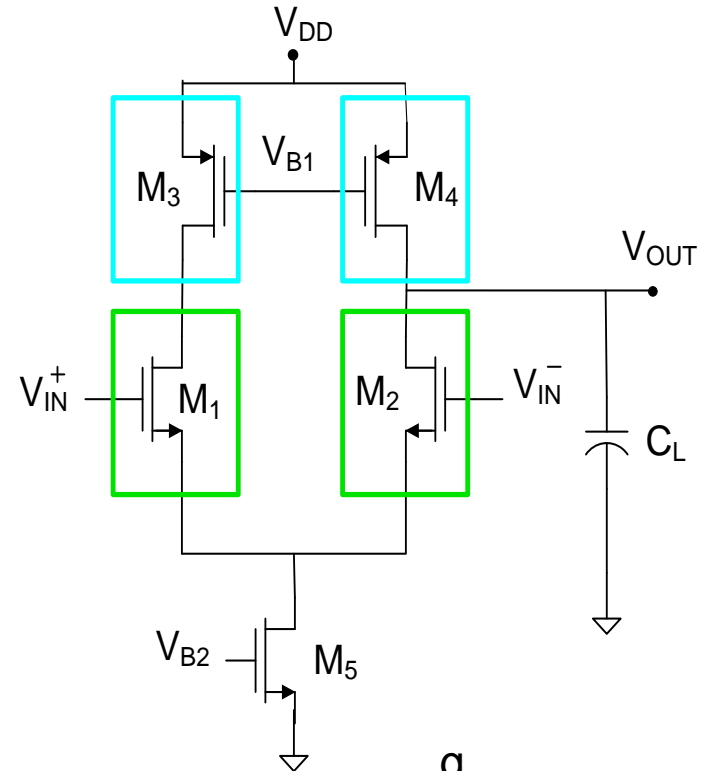-- The "differential" gain --
Need CMFB circuit
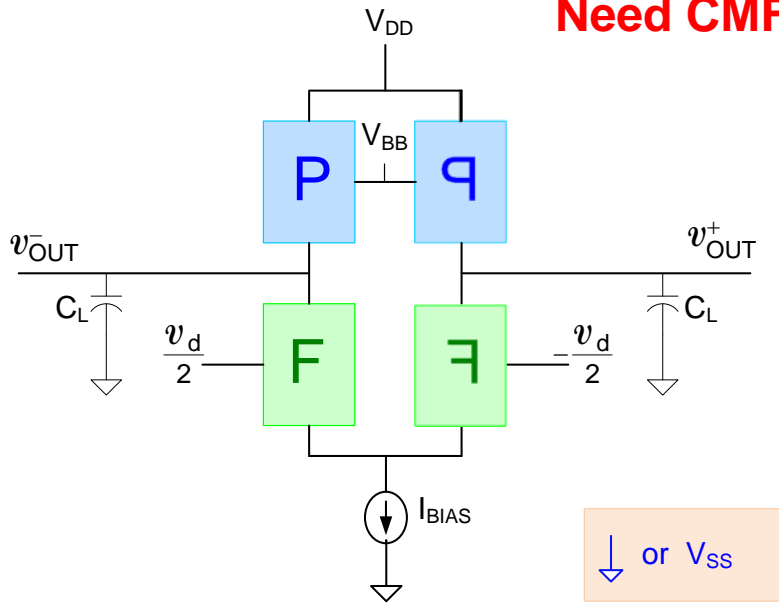
$$A(s) = \frac{v_{OUT}^-}{v_d} = \frac{-\dfrac{G_M}{2}}{sC_L + G_1 + G_2}$$

$$A_{V0} = \frac{-G_{M1}}{2(G_1 + G_2)}$$

$$BW = \frac{G_1 + G_2}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$

$$A(s) = \frac{-\dfrac{g_{m1}}{2}}{sC_L + g_{o1} + g_{o3}}$$

$$GB = \frac{g_{m1}}{2C_L}$$

$$A_o = \frac{\dfrac{g_{m1}}{2}}{g_{o1} + g_{o3}}$$

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Have obtained differential gain of 5T Op Amp by inspection from quarter circuit

- General Differential Analysis
- 5T Op Amp from simple quarter circuit
- Biasing with CMFB circuit
→ Common-mode and differential-mode analysis
- Common Mode Gain
- Overall Transfer Characteristics

Stay Safe and Stay Healthy !

End of Lecture 3